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Gradient-index nanophotonics and metamaterials

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ABSTRACT

It is commonly believed that electromagnetic waves cannot propagate in lossy conductive media and that they quickly decay inside such media over short length scales of the order of the so-called skin depth. Here we prove that this common belief is incorrect if the conductive medium is stratified. We demonstrate that electromagnetic waves in stratified lossy conductive media may have propagating character, and that the propagation length of such waves may be considerably larger than the skin depth. Our findings enable novel electromagnetic metamaterial designs by mediating the effect of losses on electromagnetic signal propagation in metamaterials. Our results demonstrate a new class of inherently non-Hermitian electromagnetic media with high dissipation, no gain, and no PT-symmetry, which nevertheless have almost real eigenvalue spectrum.

Keywords: surface electromagnetic wave, nanophotonics, metamaterial

1. INTRODUCTION

From the point of view of electromagnetic theory, all non-magnetic media can be sorted out into two broad categories, such as transparent dielectric (or non-conductive) media, which transmit electromagnetic waves, and conductive media, which are commonly believed to disallow electromagnetic wave propagation below their plasma frequency. It is generally assumed that electromagnetic waves quickly decay inside lossy conductive media over short length scales of the order of the so-called skin depth. We are going to demonstrate that this common belief is incorrect if the conductive medium is stratified, even if both components of the stratified medium are strongly lossy and conductive.

Stratification occurs naturally in many conductive media under the influence of gravity. Examples of such stratification include many underground sedimentary rocks and soils [1], seawater layer on top of sandy seabed, and many other terrestrial and astronomical [2] settings. Biological tissues are also often stratified into layers of different electric conductivity (for example, skull bone and grey matter [3]). Artificial stratification is also often implemented in various electromagnetic metamaterial structures, which typically exhibit very high losses [4]. Therefore, our surprising results on long-distance electromagnetic wave propagation in strongly lossy conductive media have broad implications in many fields of physics and engineering. As we will discuss below, our results also complement recent observations of loss-enhanced transmission due to PT-symmetry in non-Hermitian optical systems [5].

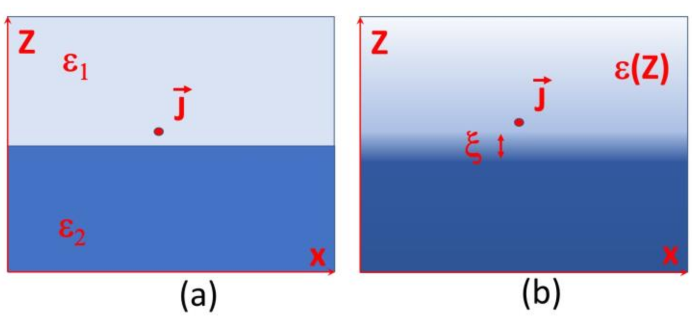

Figure 1. Geometries of the problems of interest. The dielectric permittivity of the medium ε depends only on z coordinate, which is illustrated by the halftones: (a) Usually considered step-like distribution of ε(z). (b) Gradual interface between two media. The transition layer thickness equals ξ.

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It is well established in the literature that sharp interfaces separating media having different electromagnetic properties (see Fig.1a) may support low-loss propagation of surface electromagnetic waves. The most well-known examples of such surface waves include surface plasmon-polaritons (SPP), which propagate along a sharp interface between a good metal and a dielectric [6], and the so-called Zenneck surface wave [7], which may exist at an interface between a highly lossy conductive medium and a good dielectric. The goal of this paper is to consider a more general situation in which the dielectric permittivity of a medium changes gradually across some planar surface, as shown in Fig.1b. I will demonstrate that a low loss propagating electromagnetic wave may be sent along such a planar surface inside a lossy medium, even if the imaginary part of medium permittivity remains very high on both sides of the surface. This new surface wave solution of the macroscopic Maxwell equations appears when the interface between two media is no longer considered to be abrupt. This surprising result is applicable to any portion of the electromagnetic spectrum from the extremely low radio frequencies (ELF) up to the visible and UV ranges.

2. RESULTS

Let us consider solutions of the macroscopic Maxwell equations in a geometry in which the medium is non-magnetic (*B=H*), the dielectric permittivity of a medium is continuous, and it depends only on *z* coordinate: $\varepsilon=\varepsilon(z)$, as illustrated in Fig.1b. Under such conditions, the spatial variables in the Maxwell equations separate, and without loss of generality we may assume electromagnetic mode propagation in the *x* direction, leading to field dependencies proportional to $e^{i(kx-\omega t)}$. The wave equation under such assumptions may be written as [3]:

$$-\nabla^2\vec{E}-\vec{\nabla}\left(E_z\frac{\partial\varepsilon}{\varepsilon\partial z}\right)=\frac{\varepsilon\omega^2}{c^2}\vec{E} \qquad (1)$$

For the $E_z=0$ (TE) polarization we obtain an effective Schrödinger equation

$$-\frac{\partial^2E_y}{\partial z^2}-\frac{\varepsilon(z)\omega^2}{c^2}E_y=-k^2E_y \quad , \qquad (2)$$

while for the $E_z\neq 0$ (TM) polarization the effective Schrödinger equation is

$$-\frac{\partial^2\psi}{\partial z^2}+\left(-\frac{\varepsilon(z)\omega^2}{c^2}-\frac{1}{2}\frac{\partial^2\varepsilon}{\varepsilon\partial z^2}+\frac{3}{4}\frac{(\partial\varepsilon/\partial z)^2}{\varepsilon^2}\right)\psi=-\frac{\partial^2\psi}{\partial z^2}+V\psi=-k^2\psi \quad , \qquad (3)$$

where the effective wave function ψ has been introduced as $E_z=\psi/\varepsilon^{1/2}$ [3]. For both polarizations $-k^2$ plays the role of effective energy in the corresponding Schrödinger equations.

Let us study solutions of Eqs.(2,3) which have a propagating wave character (Im(*k*)<<Re(*k*)). In the case of TE polarized light (see Eq. (2)), the effective potential energy is $V(z)=-\frac{\varepsilon(z)\omega^2}{c^2}$, and there are no surface wave solutions. Eq.(2) only admits propagating solutions described by planar waveguide-like distributions of $\varepsilon(z)$, in which the dielectric permittivity is positive and almost pure real. On the other hand, the TM polarized solutions of Eq. (3) may be much more interesting. For the TM polarization the effective potential energy near a gradual interface between two media shown in Fig.1b may be written as

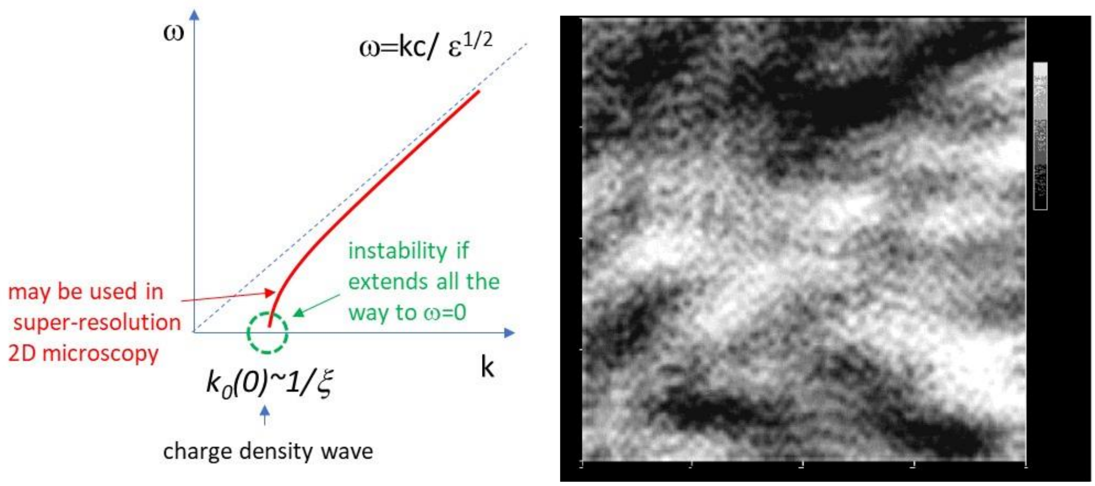
$$V(z)=-\frac{4\pi^2\varepsilon(z)}{\lambda_0^2}-\frac{1}{2}\frac{\partial^2\varepsilon}{\varepsilon\partial z^2}+\frac{3}{4}\frac{(\partial\varepsilon/\partial z)^2}{\varepsilon^2} \quad , \qquad (4)$$

where λ_0 is the free space wavelength. If $\varepsilon(z)$ of the medium changes across the interface on the spatial scale ξ, and this spatial scale is much smaller than λ_0, the second and the third term will dominate in Eq.(4). Moreover, if these terms are engineered (by either nanofabrication or by suitable material choice) in such a way that Im(*V*)<<Re(*V*), and the resulting potential well is deep enough, the wave vector of the resulting surface wave solution will be very large ($k\sim 1/\xi>>2\pi/\lambda_0$)

and this surface wave will have a propagating character (Im(k)<<Re(k)). Based on these arguments and Eq.(3), the dispersion law of this surface wave is shown schematically in Fig.2a. Note that a gradient-index medium which is used to support such a propagating surface wave solution does not need to be a low-loss medium. For example, a medium having pure imaginary dielectric permittivity $\varepsilon(z)=i\varepsilon''(z)=i\sigma(z)/\varepsilon_0\omega$ (where ε_0 is the dielectric permittivity of vacuum, and the medium conductivity $\sigma(z)$ is expressed in practical SI units) will still result in Im(V)<<Re(V):

$$V=-\frac{i\sigma\omega}{\varepsilon_0 c^2}-\frac{1}{2}\frac{\partial^2\sigma}{\sigma\partial z^2}+\frac{3}{4}\frac{\left(\partial\sigma/\partial z\right)^2}{\sigma^2} \tag{5}$$

The second and third terms in Eq. (5) are real, and they are much larger than the first term, if once again we assume that $\xi<<\lambda_0$.



(a) (b)

Figure 2. (a) Dispersion law of the TM-polarized surface waves described by Eq.(3). Note that the range of wave vectors $k\sim1/\xi>>2\pi/\lambda_0$ may be used in super-resolution microscopy. (b) 4μmx4μm near field optical image of a charge density wave in $Bi_{0.3}Ca_{0.7}MnO_3$ [11].

We should note that the physical origins of the newly predicted surface wave solutions are different from the origins of surface plasmons. Indeed, these novel surface wave solutions may be traced back to the well-known effect of charge accumulation whenever there is a gradient of conductivity in a medium and a non-zero component of electric field parallel to it. In the low frequency (electrostatic) limit the corresponding volumetric charge density ρ is obtained as

$$\rho=-\frac{\varepsilon_0\nabla\sigma\cdot\vec{E}}{\sigma} \tag{6}$$

(see for example [8]). While this effect disappears (ρ=0) inside a homogeneous conductive medium, the charges may accumulate near a planar surface inside a lossy conductive medium, if the medium conductivity changes continuously across such surface. This charge accumulation may start to vary over time, leading to appearance of relatively low-loss dynamic wave of charge density propagating along such surface. In fact, charge density waves are well known in various fields of electromagnetism and solid-state physics, which range from superconductivity [9] to ELF underwater communication [10]. Most experimental observations of charge density waves occur in static situations [11], which basically correspond to the $\omega\rightarrow0$ limit of Eq.(3) (see Fig. 2b), but propagating charge density waves have been observed too (see for example [12]).

In general, solutions of the effective Schrödinger Equation (3) with an effective potential $V(z)$ given by Eq. (4) must be obtained numerically. However, these equations may be solved analytically for some simple spatial distributions of $\varepsilon(z)$ [13]. For example, such an analytical solution has been obtained for a simple parabolic spatial distribution of the dielectric permittivity inside a single conductive stratum:

$$\varepsilon(z) = A + Bz^2 , \tag{7}$$

where both A and B are large imaginary coefficients, so that $\alpha^2=A/B$ is real and positive. These assumptions are typically valid for graphite and silicon in the UV range [14] and for seawater and many biological tissues in the radio frequency range [3]. The resulting effective potential for the TM electromagnetic wave is

$$V(z) \approx -\frac{4\pi^2 A}{\lambda_0^2}\left(1+\frac{z^2}{\alpha^2}\right) - \frac{1}{z^2+\alpha^2} + \frac{3z^2}{\left(z^2+\alpha^2\right)^2} , \tag{8}$$

where λ_0 is the free space wavelength. Analytical analysis of this effective potential, which is described in detail in [13] produces the following result for the ratio of the surface wave propagation length L and the skin depth δ in the center of the stratum (where $\varepsilon=A$):

$$\frac{L}{\delta} = \frac{3\lambda_0}{7\pi\left|A\right|^{1/2}\alpha} \approx \frac{\lambda}{7\alpha} , \tag{9}$$

which means that the TM wave propagation length may indeed be much larger than the skin depth if $\lambda>>\alpha$. Moreover, the obtained TM wave has propagating character. Its wavelength λ_{TM} calculated as

$$\lambda_{TM} = \frac{2\pi}{\mathrm{Re}(k)} \approx 4\sqrt{2}\pi\alpha \tag{10}$$

appears to be much smaller than L in the $\lambda>>\alpha$ limit. As was also noted in [13], a multilayer metamaterial based on periodically replicated parabolic spatial distribution of the dielectric permittivity (7) will behave as an artificial high refractive index metamaterial. Such novel surface electromagnetic waves and artificial high index metamaterials may be used in novel super-resolution microscopy schemes. As illustrated by Fig.2, such waves appear to have finite wave vector in the $\omega\rightarrow 0$ limit, which means that resolution of such microscopy techniques may exceed the Abbe's $\lambda_0/2$ limit by many orders of magnitude. In fact, in many situations λ_{TM} appears to approach the atomic scale [15].

3. CONCLUSION

In conclusion, we have demonstrated that contrary to common beliefs, electromagnetic waves may propagate through some lossy conductive media. Typically, such media are characterized by non-Hermitian Hamiltonians having complex eigenvalue spectrum, which results in strong wave attenuation. It has been recently demonstrated however that a non-Hermitian system may have real eigenvalue spectrum if such a system exhibits PT-symmetry [16]. Usually, the real spectrum leading to long-range wave propagation in such systems is enabled by engineered counterplay between loss

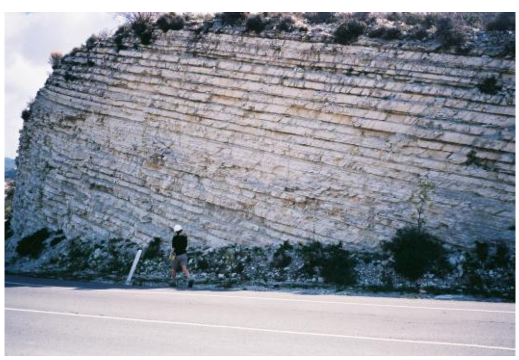


Figure 3. Example of a conductive layered sedimentary rock.

and gain [5]. The fundamental importance of our findings consists in the demonstration of a new class of electromagnetic media with inherently high dissipation, no gain and no PT-symmetry, which nevertheless has almost real eigenvalue spectrum, and which supports propagating electromagnetic waves. The non-trivial non-perturbative character of these newly found propagating electromagnetic waves is revealed by the fact that unlike conventional wave-like solutions of source-free Maxwell equations, these waves completely disappear in the lossless limit. The very existence of these newly found highly non-trivial electromagnetic waves depends on the presence of very high losses ($\varepsilon'' >> \varepsilon'$).

Our findings have broad implications in many fields of science and engineering. They enable radio communication and imaging through conductive media, such as seawater [17], various soils, plasma and biological tissues [3]. For example, since natural stratification is often observed in many underground sedimentary rocks and soils (see Fig.3), and these rocks and soils typically have $\varepsilon'' >> \varepsilon'$ at radio frequencies, using our theoretical results it may become possible to greatly improve spatial resolution and ground penetrating performance of the ground penetrating radar (GPR) techniques. Our findings may also enable novel super-resolution microscopy techniques and lower-loss electromagnetic metamaterial designs working across such previously inaccessible frequency ranges as deep UV, in which all the conventional optical materials suffer from very large losses [14].

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